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Thermal Radiation of Reissner-Nordström Black Hole

Qing-Miao Meng · Shuai Wang · Zhong-Rang Li

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Abstract By the entropy density near the event horizon, the result has been obtained that the thermal radiation of the black hole satisfies the generalized Stenfan-Boltzmann law. The derived generalized Stenfan-Boltzmann coefficient is no longer a constant, but a proportional coefficient related to the black hole mass, the black hole charge, the average radial effusion velocity of the radiation particles near the event horizon, the cut-off distance and the thin film thickness. For an extreme Reissner-Nordström black hole, radiation energy flux and radiation power are all equal to zero. Then, the generalized Stenfan-Boltzmann law will lead to a black hole remnant. In this paper, we have put forward a thermal particle model in curved space-time. By this model, the thermal radiation of the Reissner-Nordström black hole has been studied. The result shows that when the thin film thickness and the cut-off distance are both fixed for the Reissner-Nordström black hole, the radiation energy flux received by observer far away from the Reissner-Nordström black hole is proportional to the average radial effusion velocity of the radiation particles, and inversely proportional to the square of the distance between the observer and the black hole.

Keywords Reissner-Nordström black hole · Average radial effusion velocity · Generalized Stenfan-Boltzmann law · Radiation energy flux

1 Introduction

In 20th century 70's, Hawking proved in theory that there exists the thermal radiation near the black hole even horizon [1, 2]. This has greatly promoted the development of the black hole thermodynamics. Many methods have been put forward to calculate the black hole entropy [3–6]. For example, the brick-wall model proposed by 't Hooft gives a statistical

Q.-M. Meng (✉) · Z.-R. Li
Department of Physics, Heze University, Heze 274015, China
e-mail: mengqingmiao@yahoo.com.cn

S. Wang
Department of Physics, Shanghai Jiao Tong University, Shanghai 200030, China

explanation to the origin of the black hole entropy [3]. In recent years, the brick-wall model has been developed to the thin film model [7–10]. According to this thin film model, the entropy of the black hole is considered coming from the contribution of quantum field in infinitesimal thin film near its event horizon. Many researchers have adopted the thin film model to calculate the black hole entropy, and obtained the same result that the black hole entropy is proportional to the area of the event horizon [11, 12]. Where the event horizon is, there are black hole entropy and Hawking radiation [13]. In 2000, considering the self-gravitation action of the radiation particles and regarding the Hawking radiation as a quantum tunnelling process, Parikh and Wilczek obtained that the quantum tunnelling rate of the radiation particles near the event horizon is related to the change of the entropy of Bekenstein-Hawking [14]. Then, some authors have obtained the same result as that of Parikh [15–17]. It is shown that there must be an intrinsic relation between the entropy and the thermal radiation of the black hole. Therefore, it is of significance to further study this intrinsic relation. Recently, we have studied the thermal radiation of the black hole by the entropy density near its event horizon, and found that the thermal radiation of the black hole satisfies the generalized Stenfan-Boltzmann law [18–25]. The corresponding generalized coefficient is no longer a constant, but a coefficient related to the space-time metric around the black hole. Therefore, the thermal radiation of the curved space-time is different from that of the flat space-time. The strong gravitational field and the electromagnetic field around the black hole will affect its thermal radiation. In order to make these results more universally significant, by the thin film model, we have studied the thermal radiation of the Reissner-Nordström black hole, and discovered an intrinsic relation between the gravitational field and the electromagnetic field around the black hole and its thermal radiation.

2 Radiation Energy Flux and Radiation Power of Reissner-Nordström Black Hole

The line element of the Reissner-Nordström black hole is given by

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where M is the black hole mass, Q the black hole charge. The radius of the inner and outer horizon are expressed by

$$r_- = M - \sqrt{M^2 - Q^2}, \qquad r_H = M + \sqrt{M^2 - Q^2} \tag{2}$$

and the temperature of the event horizon

$$T = \frac{r_H - r_-}{4\pi r_H^2}. \tag{3}$$

By the thin film model, the statistical entropy of Dirac field of the static spherically symmetric black hole is [26]

$$S = \frac{7\pi^2 \delta A_H}{45\beta_H^3 \varepsilon(\varepsilon + \delta) f^2(r)|_{r=r_H}}, \tag{4}$$

where $\beta_H = \frac{1}{T}$, $A_H = 4\pi r_H^2$ is the area of the event horizon, ε is the cut-off distance and δ is the thickness of the thin film, and $f(r)$ satisfies the equation $g_{00} = f(r)(r - r_H)$, $g_{00} = g_{tt}$.

For the Reissner-Nordström black hole, $f(r)|_{r=r_H} = \frac{r_H - r_-}{r_H^2}$. Therefore, one can obtain the statistical entropy of Dirac field of the Reissner-Nordström black hole

$$S = \frac{7\pi^2 \delta A_H r_H^4}{45\varepsilon(\varepsilon+\delta)(r_H - r_-)^2} T^3. \tag{5}$$

According to the thin film model, the entropy density of Dirac field near the event horizon can be obtain from (5)

$$s = \frac{S}{V} = \frac{8\pi^2 M^2}{45\beta^3 \varepsilon(\varepsilon+\delta)}, \tag{6}$$

where V is the volume of the thin film near the event horizon. Since the value of r_H is much greater than that of ε and δ, the volume of the thin film can be adopted as $V = 4\pi r_H^2 \delta$.

Based on the local equivalence principle, in the local area of the infinitesimal thin film near the event horizon, the basic equations of thermodynamics still hold. The energy density ρ, the entropy density s in the thin film near the event horizon, and the temperature in local area T satisfy the following relation [18]

$$\rho = bT^4, \tag{7}$$

$$s = \frac{4}{3} bT^3. \tag{8}$$

Combining (6) and (8), one can get

$$b = \frac{7\pi^2 r_H^4}{60\varepsilon(\varepsilon+\delta)(r_H - r_-)^2}. \tag{9}$$

Substituting (9) into (7), one can obtain

$$\rho = \frac{7\pi^2 r_H^4}{60\varepsilon(\varepsilon+\delta)(r_H - r_-)^2} T^4. \tag{10}$$

It can be seen that, for a black hole, the energy density of Dirac field in the thin film is proportional to the quartic of the temperature of its event horizon when the cut-off distance and the thickness of thin film are fixed. Considering the physical mechanism of Hawking radiation of the black hole, due to the virtual particle pairs caused by the vacuum fluctuation near the event horizon, the energy of the black hole will decrease when the virtual particles with negative energy go back to the black hole by the tunnelling effects. At the same time, the virtual particles with positive energy will emit out of the gravitational area of the black hole and fly far away to form Hawking radiation. In fact, the motion of the particles with positive energy is very complex in the thin film ($r_H + \varepsilon \to r_H + \varepsilon + \delta$). The world line of the particles of zero rest mass is kind of light, while that of the particles of rest mass is kind of time. In order to make this question simple, we assume that the average radial effusion velocity of the radiation particles with positive energy is

$$v_e = \bar{v}_r^{(+)} = \int_0^\infty v_r f(v_r) \mathrm{d}v_r, \tag{11}$$

where the v_r is the radial velocity of the particles with positive energy, $f(v_r)$ is the radial velocity distribution function of the particles with positive energy, and the superscript (+)

expresses the integral range $v_r > 0$. The value of v_e is not only related to the kinds of radiation particles, but also space-time metric near the event horizon. The greater the black hole mass is, the stronger the gravitational field near its event horizon is, the smaller the value of v_e is. The gravitational field near the event horizon is extremely strong, so the value of v_e is very small. The average time which it takes for the particles with positive energy to reach the radiation spherical area of the black hole $(4\pi(r_H + \varepsilon + \delta)^2)$ is expressed by

$$\bar{t} = \frac{\delta\lambda}{2v_e}, \tag{12}$$

where λ is a revised constant. Combining (10) and (12), one can obtain the radiation energy flux of the Dirac field near the black hole event horizon.

$$M_h^{(D)} = \frac{\rho V}{A\bar{t}} = \frac{4\pi^2 M^2 v_e}{15\varepsilon(\varepsilon+\delta)\lambda} T^4. \tag{13}$$

According to (2), let

$$\sigma = \frac{7\pi^2 r_H^4 v_e}{30\lambda\varepsilon(\varepsilon+\delta)(r_H - r_-)^2} = \frac{7\pi^2(M+\sqrt{M^2-Q^2})^4 v_e}{120\lambda\varepsilon(\varepsilon+\delta)(M^2-Q^2)}, \tag{14}$$

then (13) can be rewritten as

$$M_h^{(D)} = \sigma T^4. \tag{15}$$

It can be seen that the radiation energy flux of the black hole is proportional to the quartic of the temperature of its event horizon when ε, δ and v_e are all fixed. Equation (15) can be called the generalized Stefan-Boltzmann law of the Reissner-Nordström black hole. And (14) corresponds to the generalized Stefan-Boltzmann coefficient. The derived σ is no longer a constant, but a proportional coefficient related to the black hole mass, the black hole charge, the average radial effusion velocity of the radiation particles near the event horizon, the cut-off distance and the thin film thickness. The greater the black hole mass is, and the smaller the cut-off distance and thin film thickness are, the more significant the vacuum fluctuation is. Then the value of σ will be greater because Hawking radiation is related to the vacuum fluctuation near the event horizon. When the average radial effusion velocity of radiation particles is greater, their ability to escape from the black hole is stronger, and then the value of σ will be larger. Because the black hole has charge, the electromagnetic field around the black hole will affect the vacuum fluctuation near the event horizon, and affect the thermal radiation. It can be seen that there must be an intrinsic relation between the thermal radiation and both stronger gravitation field and electromagnetic field around the black hole.

When $M = Q$, a Reissner-Nordström black hole evolves into an extreme Reissner-Nordström black hole. From (2) and (3), one can easily obtain $r_H = r_- = M$, then the inner and the outer horizon is the same. Therefore, one-way membrane region reduces to infinitesimal thin film. And the temperature of the event horizon of the extreme Reissner-Nordström black hole is equal to zero. By using (2), (3) and (13), one can obtain the radiation energy flux of the extreme Reissner-Nordström black hole

$$M_h^{(D)} = \frac{7v_e(M^2-Q^2)}{1920\pi^2\lambda\varepsilon(\varepsilon+\delta)(M+\sqrt{M^2-Q^2})^4}. \tag{16}$$

It can be seen that the radiation energy flux of the extreme Reissner-Nordström black hole is equal to zero. There is not thermal radiation for the extreme Reissner-Nordström black hole. By the derived generalized Stefan-Boltzmann law, one can infer that the Reissner-Nordström black hole will not disappear due to Hawking radiation, and there should be a black hole remnant in the end. Therefore, this will lead to two important results. The first one is that there is not the problem of information loss in black hole evaporation [27]. The second one is that the dark matter may come from the black hole remnant.

3 Radiation Energy Flux Received by Observer Far Away from Reissner-Nordström Black Hole

In order to study the effects of the gravitational field and the electromagnetic field around the radiation source in curved space-time on its thermal radiation, motivated by the particle model in classical mechanics, we have introduce a thermal particle model of the radiation source in curved space-time [24, 25]. For a radiation source with the mass M, charge Q and the radiation temperature T, if its dimension is much smaller than other dimensions involved in research issues, that is to say, its size and shape are non-functional or only play a secondary role, this radiation source can be considered as a geometric point without size and shape, but it holds the mass, the charge and temperature of the whole radiation source. And then, we call such a geometric point a thermal particle. Although the thermal particle is only an ideal model, it can bring much convenience for us to study the thermal radiation of the object in curved space-time. For example, the distant star and the static spherically symmetric black hole can be considered as thermal particles under special circumstance. One can further study their thermal radiation and discover the effects of the gravitational field and electromagnetic field around them on their thermal radiation.

For the Reissner-Nordström black hole far away from an observer, it can be considered as a thermal particle. Then, from (13) and (14), one can obtain that the radiation energy flux received by the observer is

$$M_r^{(D)} = \frac{7\pi^2(M+\sqrt{M^2-Q^2})^6 v_r}{120\varepsilon(\varepsilon+\delta)(M^2-Q^2)r^2}T^4, \tag{17}$$

were r is the distance between the observer and the black hole. One can verify the following formulas by dimensional analysis

$$l_p = \sqrt{\frac{\hbar G}{c^3}}, \qquad m_p = \sqrt{\frac{\hbar c}{G}}, \qquad T_p = \sqrt{\frac{c^5 \hbar}{G k_B^2}}, \qquad \sigma_p = \frac{k_B^4}{\hbar^3 c^2}, \tag{18}$$

where l_p is the Planck length, m_p the Planck mass, T_p the Planck temperature, σ_p the Planck generalized Stefan-Boltzmann coefficient, $\hbar$ the Planck constant, G the gravitational constant, c the light speed in vacuum, k_B the Boltzmann constant. By (18), (14) can be written in the common unit system

$$\sigma = \frac{7\pi^2\sigma_p}{120c^5}\frac{[GM+\sqrt{(GM)^2-GQ^2}]^4 v_r}{\varepsilon(\varepsilon+\delta)[(GM)^2-GQ^2]}. \tag{19}$$

By (18), (3) can be written in the common unit system

$$T = \frac{\hbar c^3}{2\pi k_B}\frac{\sqrt{(GM)^2-GQ^2}}{[GM+\sqrt{(GM)^2-GQ^2}]^2}. \tag{20}$$

Combining (19) and (20), (17) can be written in common unit system

$$M_r^{(D)} = \frac{7\hbar c}{1920\pi^2} \frac{[(GM)^2 - GQ^2]v_r}{\varepsilon(\varepsilon+\delta)r^2[GM + \sqrt{(GM)^2 - GQ^2}]^2}. \tag{21}$$

It can be seen that when the cut-off distance and the thin film thickness for the Reissner-Nordström black hole are both fixed, the radiation energy flux of the black hole is proportional to the average radial effusion velocity of the radiation particles, and inversely proportional to the square of the distance between the observer and the black hole. When $GM^2 = Q^2$, a Reissner-Nordström black hole evolves into an extreme Reissner-Nordström black hole. From (21), one can obtain that the radiation energy flux the extreme Reissner-Nordström black hole is equal to zero. This due to that the temperature of the horizon of the extreme Reissner-Nordström black hole is equal to zero. This result is consistent with the known theory.

From (21), the radiation power of Dirac field of the Reissner-Nordström black hole is

$$P^{(D)} = \frac{7\hbar c^3}{480\pi} \frac{[(GM)^2 - GQ^2]v_r}{\varepsilon(\varepsilon+\delta)[GM + \sqrt{(GM)^2 - GQ^2}]^2}. \tag{22}$$

It can be seen that when the cut-off distance and the thin film thickness are both fixed, the radiation power of the Reissner-Nordström black hole is proportional to the average radial effusion velocity of the radiation particles. For an extreme Reissner-Nordström black hole, from (22) its radiation power is equal to zero.

4 Conclusions

In summary, by the entropy density near the event horizon, the thermal radiation of the black hole is studied and the result is obtained that thermal radiation of the black hole satisfies the generalized Stefan-Boltzmann law. The derived generalized Stefan-Boltzmann coefficient is no longer a constant, but a proportional coefficient related to the black hole mass, the black hole charge, the average radial effusion velocity of the radiation particles near the event horizon, the cut-off distance and the thin film thickness. This research discovers the intrinsic relation between the gravitational field and the electromagnetic field around the black hole and its thermal radiation. For an extreme Reissner-Nordström black hole, its radiation energy flux and radiation power are all equal to zero. Then, the generalized Stenfan-Boltzmann law will lead to a black hole remnant. The thermal particle model is put forward, and adopting this model the radiation law of the Reissner-Nordström black hole is studied. These results show that when the cut-off distance and the thin film thickness for the Reissner-Nordström black hole are both fixed, the radiation energy flux received by the observer far away from the Reissner-Nordström black hole is proportional to the average radial effusion velocity of the radiation particles in thin film, and inversely proportional to the square of the distance between the black hole and the observer. Although one can obtain more thermal properties of the black hole by the thin film model, the cut-off factor cannot be avoided. The paper puts forward a new method to study the thermal radiation law of the black hole.

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